LAPORAN AKHIR

PENELITIAN UNGGULAN PERGURUAN TINGGI (P)

JUDUL

PENGEMBANGAN ALGORITMA DISAGGREGASI HUJAN *SPATIO-TEMPORAL* BAYESIAN SEBAGAI INPUT MODEL SIMULASI HIDROLOGI TERDISTRIBUSI SPASIAL

Tahun ke 1 dari rencana 2 tahun

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ABSTRAK

Kebutuhan utama dalam mensimulasi banjir dengan model hidrologi terdistribusi spasial adalah data hujan dengan resolusi tinggi terdistribusi yang mewakili seluruh kejadian hujan di DAS. Kenyataan di lapangan menunjukkan bahwa data hujan dengan resolusi tinggi sangat sedikit (misal: pada DAS Sampean hanya tersedia tiga alat pengukur hujan resolusi tinggi dan yang tersedia alat pengukur hujan data hujan harian), sehingga model hujan aliran terdistribusi tidak dapat diaplikasikan. Oleh karena itu, diperlukan suatu inovasi untuk perbaikan input data dan mengurangi ketidakpastian model. Salah satu metode yang dapat digunakan untuk memperbaiki input data pada skala waktu rendah (resolusi tinggi) adalah metode disagregasi. Metode disagregasi didefinisikan sebagai suatu metode/proses pembangkitan data sintetik yang melibatkan dua skala waktu (skala waktu tinggi dan rendah) yang mensyaratkan bahwa deret sintetik skala waktu rendah harus konsisten dengan deret observasi skala waktu tinggi. Metode disagregasi pada data yang melibatkan informasi lokasi (*space*) dan waktu (*time*) disebut sebagai disagregasi *spatio-temporal*. Disagregasi *spatio-temporal* melibatkan korelasi/kebergantungan *spatial* disamping korelasi *temporal*. Kebergantungan *spatio-temporal* dapat dinyatakan dengan model *state-space*. Model *state-space* dapat digunakan untuk pemodelan hujan yang menggambarkan karakteristik kejadian dan jumlah (tinggi) hujan secara bersama-sama. Model *state-space* merupakan model yang kompleks jika melibatkan banyak parameter dan digunakan pada data berdistribusi *skewed* dan *intermittent*. Oleh karena itu, penyelesaian model *state-space* didekati dengan Bayesian melalui *Markov Chain Monte Carlo* (MCMC) dan *Gibbs Sampler*. Model ini dapat menghasilkan sifat-sifat statistik (autokorelasi, korelasi silang) data bangkitan yang baik sesuai dengan data observasi. Namun demikian model ini belum mampu menghasilkan data bangkitan pada skala waktu level rendah. Penelitian ini bertujuan untuk mengembangkan algoritma disagregasi data curah hujan *spatio-temporal* untuk menghasilkan data skala waktu rendah (per-jam) melalui data simulasi. Validasi model dilakukan dengan membandingkan karakteristik dan pola curah hujan per-jam observasi dan sintetik hasil simulasi. Model dikatakan baik, apabila karakteristik dan pola curah hujan per-jam sintetik mendekati curah hujan per-jam observasi. Algoritma yang dikembangkan dapat menghasilkan data curah hujan sintetik harian yang serupa dengan observasi harian di lokasi data *testing*. Hal ini didukung oleh hasil uji t antara data curah per-jam rata-rata antara observasi dan sintetik di stasiun hujan Maesan (T_{test}=0,00; P-Value = 1,000), yang menunjukkan tidak ada perbedaan rata-rata secara statistik. Hasil ini juga didukung oleh nilai MSE kombinasi model Bayesian *state-space* dan transformasi *adjusting* yang kecil (MSE = 0,0073).

Kata Kunci: disagregasi *spatio-temporal*, *state-space*, Bayesian, MCMC, *Gibbs sampler*

ABSTRACT

The main needs in flood simulate the hydrological model is spatially distributed rainfall data with high resolution that represent the entire distributed rainfall in the watershed. Reality on the ground shows that the rainfall data with high resolution very little (eg watershed Sampean only available on three high-resolution rain gauges and rain gauges are available daily rainfall data), so the model of a distributed flow of rain can not be applied. Therefore, we need an innovation for improved data input and reduce uncertainty models. One method that can be used to improve the input data on the low time scale of the low is a method of disaggregation. Disaggregation method is defined as a method / synthetic data generation process which involve two time scale (time scale of high and low) which requires that all of the synthetic low timescale should be consistent with the scale of observation time series high. The method involves the disaggregation of the data location information (space) and time (time) is referred to as spatio- temporal disaggregation. Spatio-temporal disaggregation involves correlation / dependence in addition to spatial temporal correlation. Spatio- temporal dependence can be expressed in state-space models. State-space models can be used to describe the characteristics of modeling rain events and the amount of (high) rain together. State-space model is a model that is complex if it involves many parameters and used on skewed distribution of data and intermittent. Therefore, the completion of state-space models can be approximated by the Bayesian Markov Chain Monte Carlo (MCMC) and Gibbs Sampler. This model can generate statistical properties (autocorrelation, cross-correlation) generation of data that fits well with the observation data. However, this model has not been able to produce data on the rise of the low-level time scales. This study aims to develop algorithms disaggregation of rainfall data to generate spatio- temporal scale of data a low time (hourly) through the simulated data. Model validation is done by comparing the characteristics and patterns of rainfall hourly observation and synthetic simulation results. Model is said to be good, if the characteristics and patterns of hourly synthetic rainfall approach hourly observation rainfall. The development algorithm can generate synthetic daily rainfall data which are similar to the daily observations at the site of testing data. This is supported by the results of the t test between hourly obervation rainfall data mean and hourly synthetic rainfall data mean (t-test = 0.00, P - Value = 1.000), which showed no difference statistically mean. This means that model is approriate for this case. This result is also supported by the small MSE value of the combination between Bayesian state-space models and adjusting transformation (MSE = 0.0073).

Keywords : spatio-temporal disaggregation, state-space, Bayesian, MCMC, Gibbs sampler

RINGKASAN

Kebutuhan utama dalam mensimulasi banjir dengan model hidrologi terdistribusi spasial adalah data hujan dengan resolusi tinggi terdistribusi yang mewakili seluruh kejadian hujan di DAS. Kenyataan di lapangan menunjukkan bahwa data hujan dengan resolusi tinggi sangat sedikit (misal: pada DAS Sampean hanya tersedia tiga alat pengukur hujan resolusi tinggi dan yang tersedia alat pengukur hujan data hujan harian), sehingga model hujan aliran terdistribusi tidak dapat diaplikasikan. Oleh karena itu, diperlukan suatu inovasi untuk perbaikan input data dan mengurangi ketidakpastian model. Salah satu metode yang dapat digunakan untuk memperbaiki input data pada skala waktu rendah (resolusi tinggi) adalah metode disagregasi. Metode disagregasi didefinisikan sebagai suatu metode/proses pembangkitan data sintetik yang melibatkan dua skala waktu (skala waktu tinggi dan rendah) yang mensyaratkan bahwa deret sintetik skala waktu rendah harus konsisten dengan deret observasi skala waktu tinggi. Metode disagregasi pada data yang melibatkan informasi lokasi (*space*) dan waktu (*time*) disebut sebagai disagregasi *spatio-temporal*. Disagregasi *spatio-temporal* melibatkan korelasi/kebergantungan *spatial* disamping korelasi *temporal*. Kebergantungan *spatio-temporal* dapat dinyatakan dengan model *state-space*. Model *state-space* dapat digunakan untuk pemodelan hujan yang menggambarkan karakteristik kejadian dan jumlah (tinggi) hujan secara bersama-sama. Model *state-space* merupakan model yang kompleks jika melibatkan banyak parameter dan digunakan pada data berdistribusi *skewed* dan *intermittent*. Oleh karena itu, penyelesaian model *state-space* didekati dengan Bayesian melalui *Markov Chain Monte Carlo* (MCMC) dan *Gibbs Sampler*. Model *state-space* diformulasikan sebagai berikut :

$$W_t(s_i) = \boldsymbol{X}_t(s_i)^T\boldsymbol{b} + \varepsilon_t(s_i) + \upsilon_t(s_i)\ ,$$

$$\boldsymbol{\varepsilon_t} = \phi . g(\boldsymbol{\varepsilon_{t-1}}) + \boldsymbol{\xi_t},\ \phi \epsilon \mathbb{R}\ ,$$

dengan $\mathbf{b}\ \epsilon \mathbb{R}^p$; $\upsilon_t(s_i)$ menyatakan *error* pengukuran; $\varepsilon_t(s_i)$ menyatakan kebergantungan *spatio-temporal*; $\upsilon_t(s_i)$ adalah iid : $\upsilon_t(s_i) \sim N(0,\tau^2)$, $\boldsymbol{\varepsilon_t} = (\varepsilon_t(s_1), \ldots, \varepsilon_t(s_n))'$, $\boldsymbol{X}_t(s_i)\ \epsilon \mathbb{R}^p$ adalah kovariat, ϕ adalah koefisien/parameter model, $g(.)$ adalah fungsi yang menentukan kebergantungan *temporal* dan $\boldsymbol{\xi_t} = (\xi_t(\boldsymbol{s_1}), \ldots, \xi_t(\boldsymbol{s_n}))'$ menghitung korelasi spasial. $\boldsymbol{\xi_t}$ diasumsikan *independent.* Model ini dapat menghasilkan sifat-sifat statistik (autokorelasi, korelasi silang) data bangkitan yang baik sesuai dengan data observasi. Namun demikian model ini belum mampu menghasilkan data bangkitan pada skala waktu level rendah. Penelitian ini bertujuan untuk mengembangkan algoritma disagregasi data curah hujan *spatio-temporal* untuk menghasilkan data skala waktu rendah (per-jam) melalui data simulasi.

Kombinasi antara model *state-space* pendekatan Bayesian dan transformasi *adjusting*, diperoleh disagregasi lokasi-waktu sebagai berikut:

$$\hat{Y}_l^s = \frac{Z_t^s}{\tilde{Z}_t^s}\tilde{Y}_l^s, \quad l = (t-1)k+1; \quad t = 1,\ldots,T; \quad s = 1,\ldots,n; \quad k = 24,$$

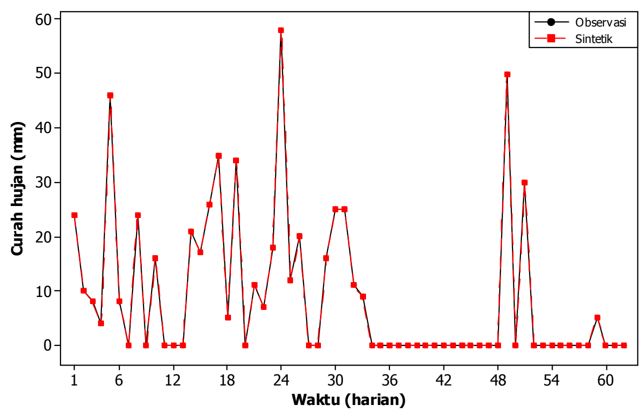
dengan $\hat{Y}_l^s$ adalah variabel skala waktu rendah setelah transformasi *adjusting* pada periode- t, subperiode- l di lokasi s, Z_t^s adalah variabel skala waktu tinggi observasi pada periode- t di lokasi s, $\tilde{Y}_l^s$ adalah variabel skala waktu rendah sebelum transformasi *adjusting* pada periode- t, subperiode- l di lokasi s yang diperoleh dari model *state-space* pendekatan Bayesian, $\tilde{Z}_t^s$ adalah variabel skala waktu tinggi sebelum transformasi *adjusting* pada

periode- t di lokasi s, yang menunjukkan jumlah dari semua $\tilde{Y}_l^s$ atau $\sum_{l=(t-1)k+1}^{tk} \tilde{Y}_l^s = \tilde{Z}_t^s$.

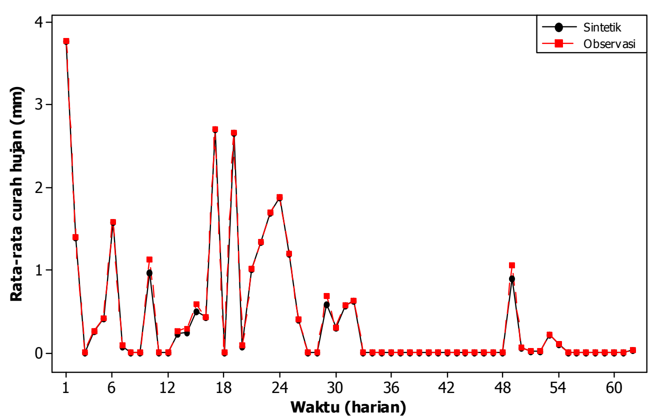
Transformasi *adjusting* dilakukan sedemikian rupa sehingga $\sum_{l=(t-1)k+1}^{k} \hat{Y}_l^s = Z_t^s$.

Validasi model dilakukan dengan membandingkan karakteristik dan pola curah hujan per-jam observasi dan sintetik hasil simulasi. Model dikatakan baik, apabila karakteristik dan pola curah hujan per-jam sintetik mendekati curah hujan per-jam observasi.

Algoritma kombinasi model *state-space* Bayesian dengan *adjusting* dapat menghasilkan data curah hujan sintetik harian yang serupa dengan observasi harian di lokasi data *testing* yaitu stasiun hujan Sukokerto (Gambar 1) dan stasiun hujan Maesan (Gambar 2). Hal ini didukung oleh hasil uji t antara data curah per-jam rata-rata antara observasi dan sintetik di stasiun hujan Maesan (T_{test}=0,00; P-Value = 1,000), yang menunjukkan tidak ada perbedaan rata-rata secara statistik. Hal ini berarti bahwa model sesuai secara statistik. Hasil ini juga didukung oleh nilai MSE kombinasi model Bayesian *state-space* dan transformasi *adjusting* yang kecil (MSE = 0.0073).



Gambar 1. Pembandingan antara nilai rata-rata harian observasi dan sintetik di stasiun hujan Sukokerto pada bulan Januari 2006 dan Januari 2007



Gambar 2. Pembandingan antara nilai rata-rata harian observasi dan sintetik di stasiun hujan Maesan pada bulan Januari 2006 dan Januari 2007

Kata Kunci: disagregasi *spatio-temporal*, *state-space*, Bayesian, MCMC, *Gibbs sampler*

SUMMARY

The main needs in flood simulate the hydrological model is spatially distributed rainfall data with high resolution that represent the entire distributed rainfall in the watershed. Reality on the ground shows that the rainfall data with high resolution very little (eg watershed Sampean only available on three high-resolution rain gauges and rain gauges are available daily rainfall data), so the model of a distributed flow of rain can not be applied. Therefore, we need an innovation for improved data input and reduce uncertainty models. One method that can be used to improve the input data on the low time scale is a method of disaggregation. Disaggregation method is defined as a method / synthetic data generation process which involves two time scale (time scale of high and low) which requires that all of the synthetic low timescale should be consistent with the high scale of observation time series. The method involves the disaggregation of the data location information (space) and time (time) is referred to as spatio- temporal disaggregation. Spatio-temporal disaggregation involves correlation/dependence in addition to spatial temporal correlation. Spatio-temporal dependence can be expressed in state-space models. State-space models can be used to describe the characteristics of modeling rain events and the amount of the high rain together. State-space model is a model that is complex if it involves many parameters and used on skewed distribution of data and intermittent. Therefore, the completion of state-space models approximated by the Bayesian Markov Chain Monte Carlo through (MCMC) and Gibbs Sampler. State-space model is formulated as follows :

$$W_t(s_i) = \boldsymbol{X}_t(s_i)^T \boldsymbol{b} + \varepsilon_t(s_i) + \upsilon_t(s_i) \ ,$$

$$\boldsymbol{\varepsilon_t} = \phi . g(\boldsymbol{\varepsilon_{t-1}}) + \boldsymbol{\xi_t}, \ \phi \epsilon \mathbb{R} \ ,$$

with $\mathbf{b} \ \epsilon \mathbb{R}^p$; $\upsilon_t(s_i)$ stated measurement error; $\varepsilon_t(s_i)$ states spatio-temporal dependence; $\upsilon_t(s_i)$ are iid : $\upsilon_t(s_i) \sim N(0, \tau^2)$, $\boldsymbol{\varepsilon_t} = (\varepsilon_t(s_1), \ldots, \varepsilon_t(s_n))'$, $\boldsymbol{X}_t(s_i) \ \epsilon \mathbb{R}^p$ are the covariates, ϕ is the coefficient/model parameters, $g(.)$ is a function that determines the temporal dependence and $\boldsymbol{\xi_t} = (\xi_t(\boldsymbol{s_1}), \ldots, \xi_t(\boldsymbol{s_n}))'$ calculate the spatial correlation. $\boldsymbol{\xi_t}$ assumed to be independent. This model can generate statistical properties (autocorrelation, cross-correlation) generation of data that fits well with the observation data. However, this model has not been able to produce data on the rise of the low-level time scales. This study aims to develop algorithms disaggregation of rainfall data to generate spatio-temporal scale of data a low time (hourly) through the simulated data.

The combination of state-space models and transformations adjusting the Bayesian approach, the location-time disaggregation is obtained as follows:

$$\hat{Y}_l^s = \frac{Z_t^s}{\tilde{Z}_t^s} \tilde{Y}_l^s, \quad l = (t-1)k+1; \quad t = 1, \ldots, T; \quad s = 1, \ldots, n; \quad k = 24,$$

with $\hat{Y}_l^s$ variable time scale is lower after adjusting transformation in period t, subperiode - l at location s, Z_t^s is the time -scale variable height observations in period t at location s, $\tilde{Y}_l^s$ is lower variable time scale before adjusting the transformation period t, subperiode - l at location s is obtained from the state-space model of the Bayesian approach, $\tilde{Z}_t^s$ is high time scale variable before adjusting transformation in period t at location s, which indicates the number of all $\tilde{Y}_l^s$ or $\sum_{l=(t-1)k+1}^{tk} \tilde{Y}_l^s = \tilde{Z}_t^s$. The adjusting transformation is done in such that

$\sum_{l=(t-1)k+1}^{k} \hat{Y}_l^s = Z_t^s$. Model validation is done by comparing the characteristics and patterns of rainfall hourly observation and synthetic simulation results. Model is said to be good, if the characteristics and patterns of hourly synthetic rainfall approach hourly observation rainfall.

Algorithm combination of Bayesian state-space models with adjusting to generate synthetic daily rainfall data are similar to the daily observations at the site of testing data that Sukokerto rainfall stations (Figure 1) and Maesan rainfall stations (Figure 2). This is supported by the results of the t test between hourly observation rainfall data mean and synthetic rainfall data mean (t-test = 0.00, P - Value = 1.000), which showed no difference statistically average. This means that model is approriate for this case. This result is also supported by the small MSE value of the combination between Bayesian state-space models and adjusting transformation (MSE = 0.0073).

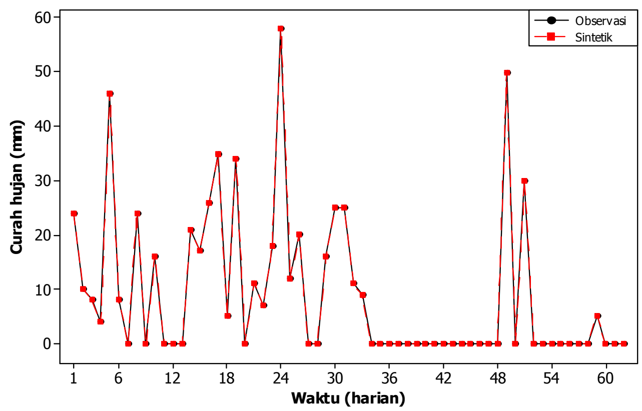


Figure 1. A comparison between the average value of daily observations and synthetic at Sukokerto rain station in January 2006 and January 2007

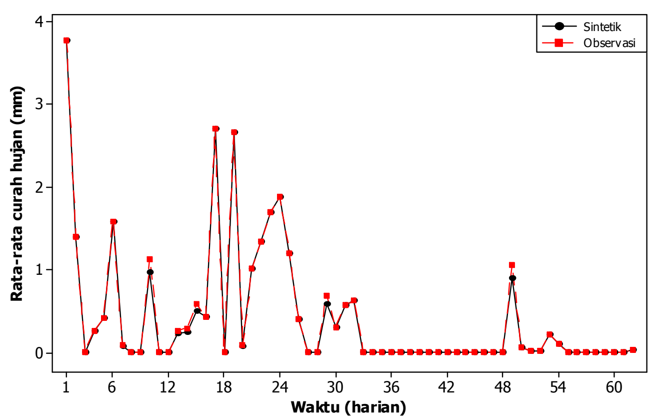


Figure 2. A comparison between the average value of daily observations and synthetic at Maesan rain station in January 2006 and January 2007

Keywords : spatio-temporal disaggregation, state-space, Bayesian, MCMC, Gibbs sampler

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